

Math at Work 11: Chapter 6

February 20, 2012, 16:01

6

Slope



Ken designs coaster rides. He gets ideas from his past projects.

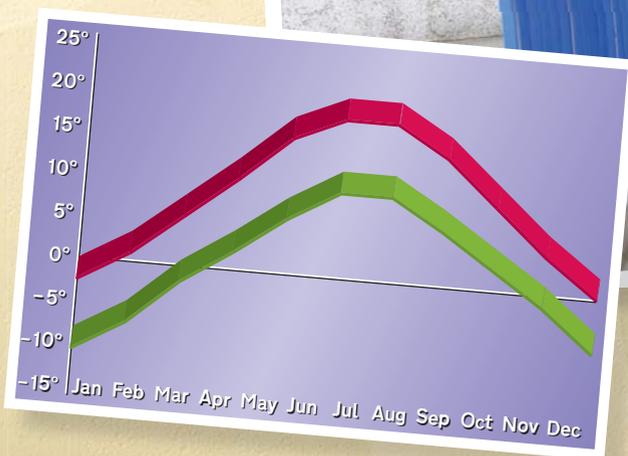
1. How does steepness affect the thrill of a ride?
2. List some differences between travelling up and down a hill on a coaster.
3. What safety factors might Ken need to consider?

Key Words

slope
rise
run
constant
angle of elevation
grade
rate of change

Career Link

Lisa is a carpenter who builds houses. There are building codes that state rules for the slopes of roofs, driveways, and stairs. Lisa checks the slope of the things she builds. She needs to make sure they meet the building code requirements.



Proportions

1. Express each fraction in lowest terms.

a) $\frac{4}{12}$

b) $\frac{8}{16}$

c) $\frac{0}{6}$

d) $\frac{7}{28}$

2. Solve.

a) $10 = \frac{x}{5}$

b) $4 = \frac{20}{x}$

c) $\frac{40}{x} = \frac{10}{3}$

d) $\frac{x}{16} = \frac{5}{4}$

3. Solve to create equivalent ratios.

a) $1:6 = \blacksquare:24$

b) $\blacksquare:12 = 6:4$

c) $30:\blacksquare = 6:5$

d) $20:1 = 100:\blacksquare$

Convert Units

4. Convert each length to inches.

a) 1 ft

b) 6'

c) 4' 5"

d) $2\frac{1}{2}$ ft

5. Convert each length to centimetres.

a) 1 m

b) 4 m

c) 8.1 m

d) 0.5 m

Percent

6. Express each decimal as a percent.

a) 0.24

b) 0.04

c) 1.25

d) 0.1

7. Express each percent as a decimal.

a) 65%

b) 44%

c) 20%

d) 1%

8. Express each fraction as a percent.

a) $\frac{61}{100}$

b) $\frac{4}{100}$

c) $\frac{5}{4}$

d) $\frac{13}{10}$

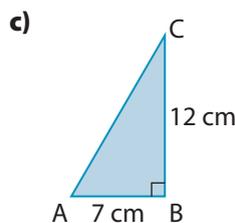
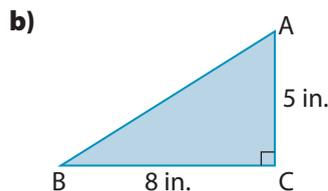
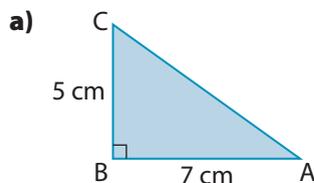
e) $\frac{14}{25}$

f) $\frac{6}{50}$

The Tangent Ratio

9. Write the tangent ratio for each $\angle A$.

$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$



10. Determine each tangent ratio. Round your answers to two decimal places.

a) $\tan 16^\circ$

b) $\tan 72^\circ$

c) $\tan 85^\circ$

On a calculator, press **C** **TAN** **16** **=** to determine the answer.

11. Determine the measure of each angle to the nearest degree. The first one is done for you.

a) $\tan A = 0.6148$

$\tan A = 0.6148$
 $\angle A = \tan^{-1}(0.6148)$

On a calculator, press

C **2nd** **TAN⁻¹** **0.6148** **=** to determine the answer.

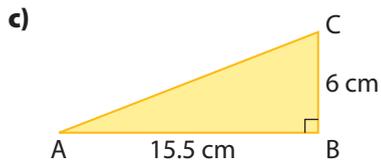
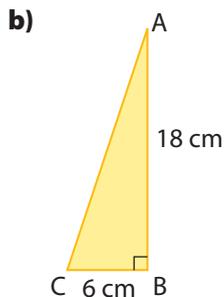
$\angle A = 31.58320053$

b) $\tan B = 0.2468$

c) $\tan C = 1.3579$

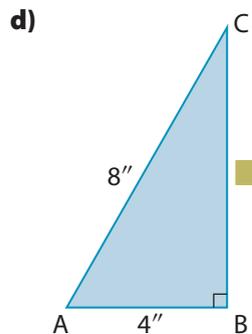
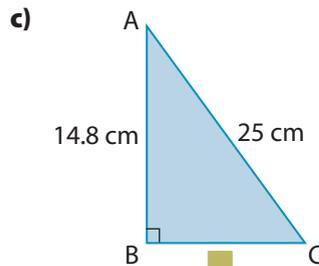
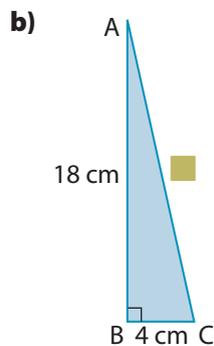
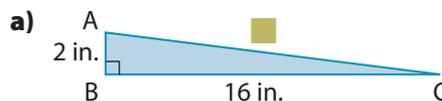
d) $\tan D = 2.7891$

12. Determine the measure of each $\angle A$ to the nearest degree. Use the tangent ratio.



The Pythagorean Relationship

13. What is the length of the indicated side in each triangle? Express your answer to the nearest unit.



6.1

What Is Slope?

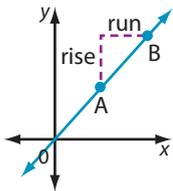
Focus On ...

- explaining slope as rise divided by run
- verifying that the slope is constant for a straight object
- solving problems that involve slope
- describing contexts that involve slope



slope

- a measure of the steepness of a line
- the ratio of the rise to the run of a line or line segment



Materials

- piece of stiff cardboard
- books
- metre stick or measuring tape
- toy car
- Explore Slope worksheet 

*Many of our actions and activities involve **slope**. Have you ever used a skateboard ramp, looked at a temperature forecast graph, or noticed a ball rolling down a driveway that you thought was level?*

Explore Slope

- Build a ramp using a piece of cardboard and some books.
 - Measure the height of the top of the ramp. Record it on the worksheet.
 - Measure and record the horizontal length of the ramp. The horizontal length goes from the bottom of the ramp to the bottom edge of the books, directly below the top of the ramp.
 - Write these values as a ratio of height to length.
 - Convert the fraction to a decimal and to a percent. Record the values.

2. a) Place a car on top of the ramp and let it go down the ramp.
- b) Measure and record the distance the car travels along the floor.



3. Raise or lower the height of the ramp. Then, repeat steps 1b) to 2b).
4. Repeat this process five times.

5. Reflect

- a) Sort the data you collected from the shortest to the longest distance travelled by the car.
- b) Record your data in a table such as the one shown.

Distance Travelled by Car (cm)	Height of Ramp Length of Ramp	
	Fraction	Percent

- c) What can you conclude about the relationship between the height of a ramp and the distance the car travels along the floor?

6. **Extend Your Understanding** The height of the top of the ramp is called the **rise**. The horizontal distance from the bottom of the ramp to the edge of the books is called the **run**. How does the ratio of the rise of the ramp to the run affect the distance travelled by the car? Explain.

Strategy



Make a Systematic List

rise

- the vertical distance between two points on a line

run

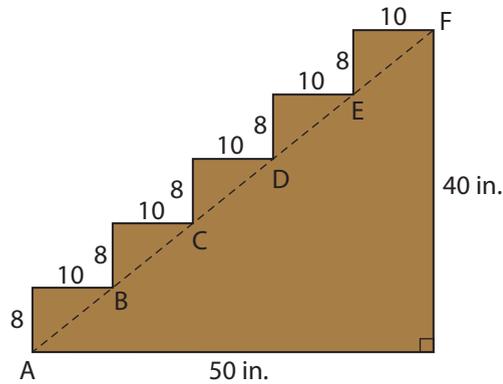
- the horizontal distance between two points on a line

On the Job 1



Calculate Slope

Aiden is a carpenter's apprentice. He sketches the side view of the stairs he is working on. Aiden needs to check that the handrail is installed at the same slope as the stairs. What should be the slope of the handrail?



Solution

Method 1: Work With the Slope of Each Step

Aiden determines the slope of each step.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The rise is the vertical distance.
The run is the horizontal distance.

$$\begin{aligned}\text{Slope of AB} &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{Slope of BC} &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{Slope of CD} &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{Slope of DE} &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{Slope of EF} &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

The slope of each step is $\frac{4}{5}$, or 4:5. The slope of the handrail must be 4:5.

constant

- a value that is always the same

Method 2: Work With the Slope of the Whole Staircase

From the diagram, it is clear that the slope of the staircase is **constant** and equal to the slope of each step. Calculate the slope of the entire staircase.

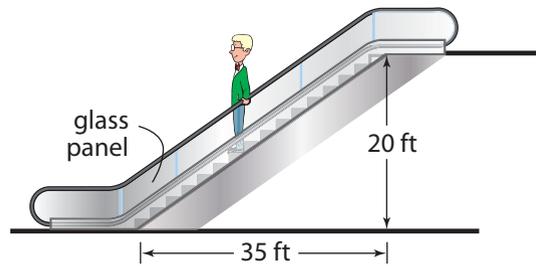
$$\begin{aligned} \text{Slope of AF} &= \frac{40}{50} \\ &= \frac{4}{5} \end{aligned}$$

Line segment AF represents the entire staircase.

The slope of the staircase is $\frac{4}{5}$, or 4:5. The slope of the handrail must be 4:5.

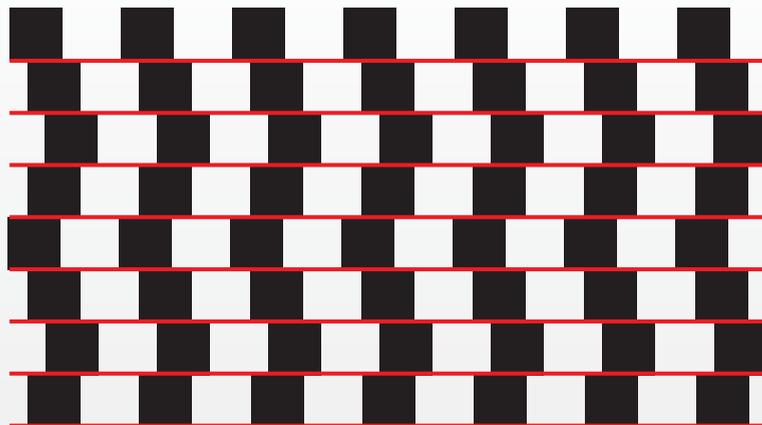
Your Turn

One panel of glass needs to be replaced on the escalator shown. Leanne places the order with the glass company, but she needs to tell them the slope. What is the slope of the glass?



Puzzler

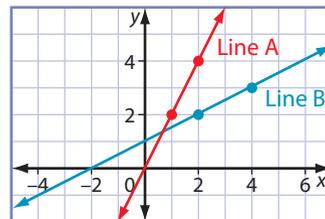
What are the slopes of the red lines?



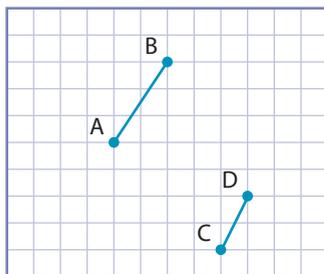
Check Your Understanding

Try It

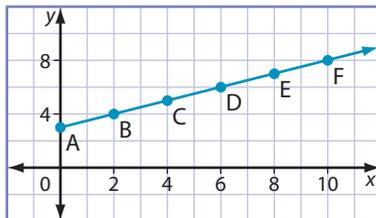
1. What are the rise and run of each line?



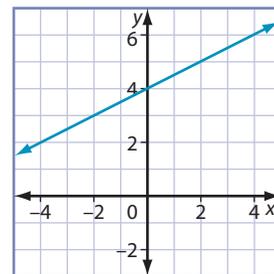
2. Determine the slope of each line segment.



3. Examine the graph.



- a) What is the slope of line segment DE?
 - b) Can you predict the slope of line segment EF? Explain.
 - c) Can you predict the slope of line segment AF? Explain.
 - d) State another line segment with the same slope as AF.
4. Determine the slope of the line shown.



5. Suppose the slope of a line is $2:3$ and the run is 6 cm. What is the rise of the line?

F.Y.I.

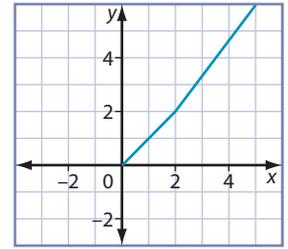
The pitch of a roof is the same as the slope of the roof. Pitch is usually expressed as a fraction, out of 12.

F.Y.I.

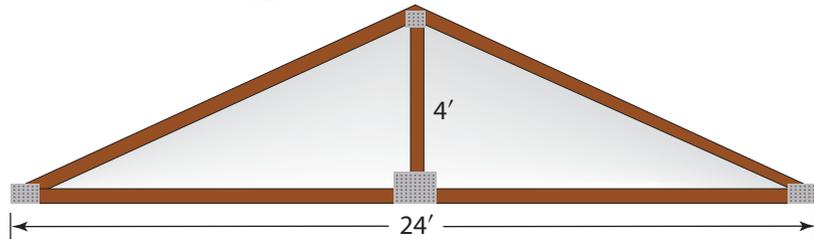
“Piano” is a shortened form of *pianoforte*, the Italian word for the instrument. *Pianoforte* was used because the sound can be loud and soft. *Piano* means soft and *forte* means loud.

**Apply It**

6. Does the graph show a straight line? Justify your answer using slope.



7. Betty says the pitch, or slope, of the roof shown is $\frac{2}{12}$. Carolyn insists the slope is $\frac{4}{12}$. Who is correct? How do you know?



8. Cole bought a piano. The delivery truck has a ramp so the piano can be easily taken off the truck. The distance from the top of the ramp to the ground is 3 ft. The ramp reaches 12 ft on the ground from the back of the truck. What is the slope of the ramp?



9. Kara and Amy are planning a ski trip to Skier’s Paradise. They check the website and see the following table.

Ski Run	Horizontal Distance	Vertical Distance
Skier’s Surprise	1576 m	519 m
Rigorous Run	419 m	220 m
Magic Mountain	231 m	95 m
Bunny Slope Express	87 m	28 m

- Calculate the slope of each run. Express each answer as a decimal to the nearest hundredth.
- Kara is an avid skier. Which run should she choose? Why?
- Amy has only skied once before and is a little nervous about skiing. Which run should she choose? Why?

On the Job 2

Solve a Problem Involving Slope

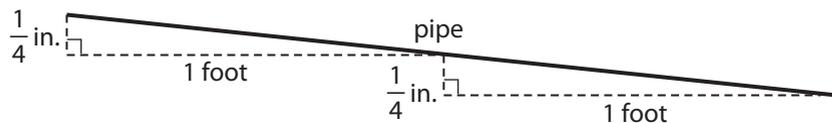
Matt's backyard has been flooding, so he decides to install a drainage pipe. Matt learns that the pipe needs to drop $\frac{1}{4}$ inch per 1-foot length.



- What does it mean that the pipe drops $\frac{1}{4}$ inch per 1-foot length?
- What is the slope of the pipe?
- For a horizontal run of 100 ft, how much lower should one end of the pipe be than the other? Express the answer in feet and inches.

Solution

- A drop of $\frac{1}{4}$ inch per 1-foot length means that for every foot along the horizontal, the pipe lowers $\frac{1}{4}$ inch from one end to the other.



- Matt needs to use the same unit for both measurements. So, he converts 1 foot to 12 inches.

$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\frac{1}{4}}{12} \\ &= \frac{\frac{1}{4} \times 4}{12 \times 4} \\ &= \frac{1}{48}\end{aligned}$$

Eliminate the fraction in the numerator by multiplying it by 4. To keep the slope equivalent, multiply the denominator by the same value, 4.

The slope of the pipe is $\frac{1}{48}$.

c) Method 1: Use a Formula

The vertical distance between the ends of the pipe is the rise.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\frac{1}{48} = \frac{\text{rise}}{100}$$

$$\frac{1}{48}(100) = \frac{\text{rise}}{100}(100)$$

$$\frac{100}{48} = \text{rise}$$

$$2\frac{4}{48} = \text{rise}$$

$$2\frac{1}{12} = \text{rise}$$

There are 12 inches in 1 foot.

So, $\frac{1}{12}$ of a foot = 1 inch.

$$2\frac{1}{12}' = 2' 1''$$

For a run of 100 ft, one end of the pipe should be 2' 1" lower than the other end.

Strategy



Look for a Pattern

Method 2: Use a Pattern

$$\text{Drops } \frac{1}{4}'' \text{ in } 1'$$

$$\text{Drops } \frac{2}{4}'' \text{ in } 2'$$

$$\text{Drops } \frac{3}{4}'' \text{ in } 3'$$

$$\text{Drops } \frac{4}{4}'' \text{ in } 4'$$

⋮

$$\text{Drops } \frac{10}{4}'' \text{ in } 10'$$

⋮

$$\text{Drops } \frac{100}{4}'' \text{ in } 100'$$

$$\frac{100}{4} = 25$$

The pipe drops 25" in 100'.

Convert 25" to feet and inches.

$$25'' = 2' 1''$$

Can you describe the pattern?

There are 12" in 1'. In 2' there are 24". So, 25" must be 2' and 1" remaining.

For a run of 100 ft, one end of the pipe should be 2' 1" lower than the other end.

Your Turn

Matt continues to install drain piping using the same slope. For a horizontal run of 200 ft, how much lower should one end of the pipe be than the other end? Determine your answer using two different methods.

Check Your Understanding

Try It

1. Solve each proportion.

a) $\frac{1}{12} = \frac{x}{372}$

b) $\frac{1}{8} = \frac{28}{x}$

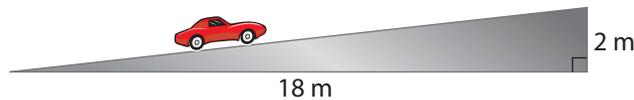
c) $\frac{x}{25} = \frac{2}{5}$

d) $\frac{75}{6} = \frac{x}{8}$

2. Copy and complete the table.

	Rise	Run	Slope
a)	60 m	10 m	
b)	15 in.	50 in.	
c)	75 cm	100 cm	
d)	1 inch	1 foot	
e)	4 inches	$2\frac{1}{2}$ feet	
f)	3 m	400 cm	
g)	50 cm	2 m	

3. Determine the slope of the ramp shown.



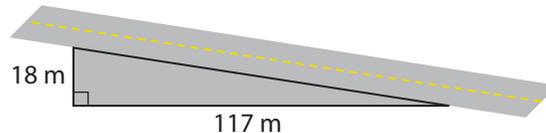
4. A ramp rises 1 ft for every 3 ft horizontally.

a) What is the slope of the ramp?

b) If the top of the ramp is 4 ft high, what is the distance along the floor?

Apply It

5. A road rises 18 m for every 117 m of horizontal distance. Determine the slope of the road.



a) Express the slope as a fraction.

b) Express the slope as a decimal.

c) Express the slope as a percent.

6. Robert built some stairs so his puppy could get up on the bed by herself. The stairs have a height of 18" and a horizontal length of 22".

a) What is the slope of the stairs?

b) What is the slope of each step? Explain.

7. Jordan assembles two ramps for his younger brother's toy cars.

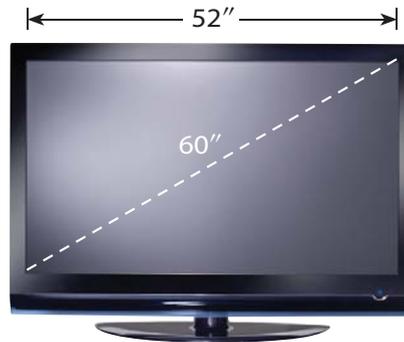
• The blue ramp is 30 cm high and has a horizontal length of 48 cm.

• The yellow ramp is 12 cm high and has a horizontal length of 14 cm.

Which ramp has the greater slope?



8. A 60-inch TV measures 52 inches wide. What is the slope of the diagonal? **Hint:** You will need to use the Pythagorean relationship.



9. Suppose a drainage pipe drops 6 in. from one end of the pipe to the other.

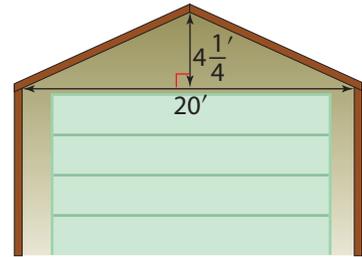
a) What is the rise of the pipe?

b) Determine the run of the pipe. Recall that a drainage pipe needs to drop $\frac{1}{4}$ inch per 1-foot length along the horizontal.

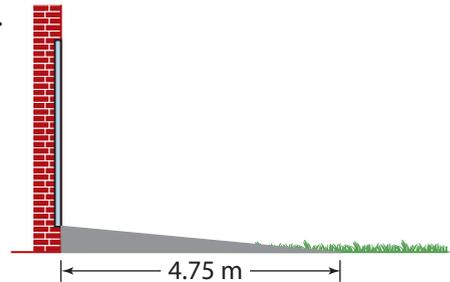
c) Drainage pipe is usually sold in lengths of 100'. How many pieces of pipe were needed for this job?

Work With It

- The garage shown is 20' wide. The vertical distance from the base of the roof to the peak is $4\frac{1}{4}$ '.
 - State the rise and run of the roof.
 - What is the pitch of the roof?
Express your answer out of 12.



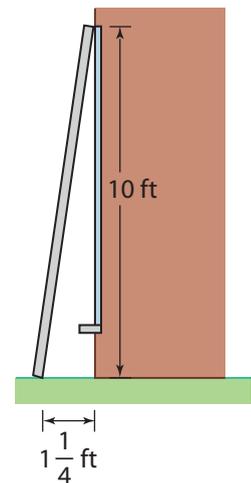
- Steve installs stone landscaping. He is building a patio that has a horizontal distance from the wall of 4.75 metres. The patio has a slight slope of $\frac{1}{95}$ down away from the house.
 - Copy the diagram of the side view of the patio. Determine the unknown lengths and add the missing labels.



- On grid paper, sketch a hill that rises 10 cm vertically for every 50 cm of horizontal run.
 - How can you check that your sketch is correct?

Discuss It

- Debbie is helping Susan clean the windows on the second floor of her house. For safety, the slope of the ladder should be close to 4:1 or 4, but not greater.
 - Susan states that the only way to find the slope of the ladder is to measure the height where the ladder touches the wall and the distance along the ground from the house to the ladder.
 - Debbie states that if the ladder is placed safely and the run is $1\frac{1}{4}$ ft, the rise must be 10 ft.



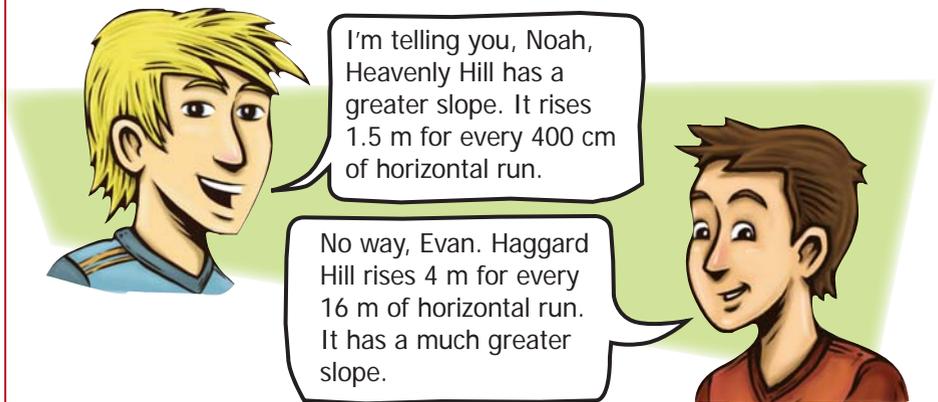
For each part, do you agree with the statement? Explain why or why not.



Tools of the Trade

Masonry workers build with stone or bricks. Masons prevent water damage inside a house by using a level and a measuring device to ensure stonework slopes away from the house. For more information on masonry, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

5. How can you verify that the slope of a line on a graph is constant?
6. Evan and Noah are discussing the slopes of two ski hills. Who is correct? Explain.



7. a) Explain what it means to say the slope of a line is 5.
b) State a possible rise and run for a line with a slope of 5. Can you give another possibility?
c) On grid paper, draw three lines that each have a slope of 5. What word could you use to describe these lines?

8. Gabrielle is planning a hiking trip. The trail consists of many small hills, some tall hills, and some flat surfaces.

- a) Identify three examples of slope that Gabrielle may see during her hike.
b) If she maintains constant speed, when will she use the most energy along the hike? Explain.
c) If she maintains constant speed, when will she use the least amount of energy? Explain.



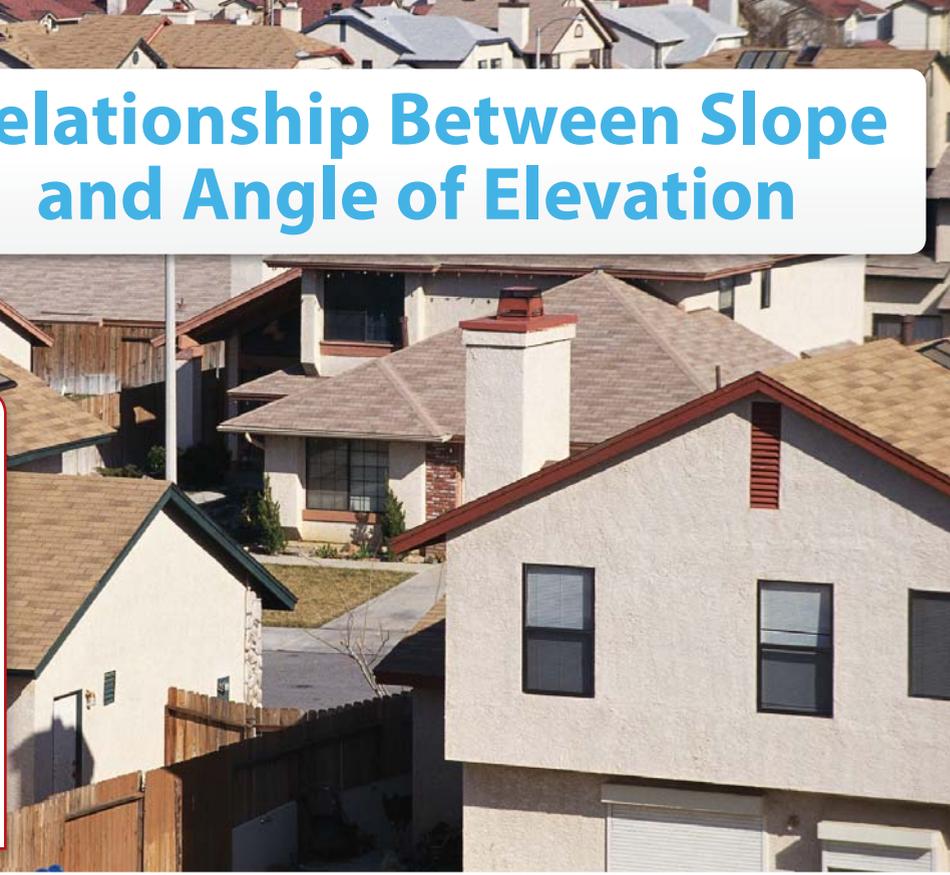
9. Explain how two hills can have the same vertical height, yet differ in slope. Use a diagram in your explanation.

6.2

Relationship Between Slope and Angle of Elevation

Focus On ...

- describing the conditions for a slope that is zero or undefined
- explaining the relationship between slope and angle of elevation
- explaining the safety implications of different slopes
- solving problems by applying the slope formula



The buildings in the picture provide many examples of surfaces with different slopes. Can you find a surface that has a large value for slope? Can you find a surface that has a slope of zero?

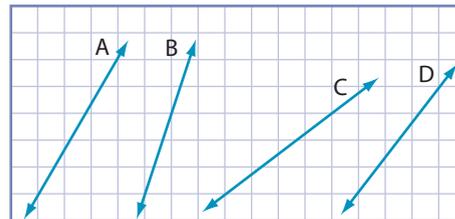
Materials

- grid paper 
- protractor
- scientific calculator
- ruler
- Explore Slope and Angle of Elevation diagrams 
- Explore Slope and Angle of Elevation worksheet 

Explore Slope and Angle of Elevation

Part 1: Connect Slope and Steepness

1. Examine lines A, B, C, and D on the grid. Just by looking, can you tell which line is the steepest? Which line is the least steep?



2. Calculate the slope of each line. Were your answers to step 1 correct? How do you know?
3. **Reflect** How could the value of the slopes help you arrange lines in order of steepness?

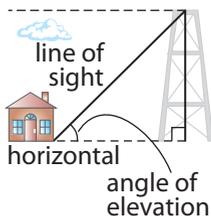
Strategy



Draw or Model

angle of elevation

- an angle formed by the horizontal and a line of sight above the horizontal line

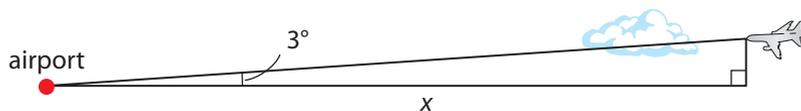


4. Extend Your Understanding

- What would a line with a slope of zero look like? On a grid, draw a line segment with a slope of zero.
 - What is the rise of this line segment? What is its run?
- Draw a vertical line segment on a grid.
 - How would you describe its slope?
 - What is the rise of the line segment? What is the run?
 - Can you determine the slope of this line?
 - What can you conclude about the slope of any vertical line?
 - What value can be assigned to the slope of any horizontal line?

Part 2: Connect Slope and Angle of Elevation

- From your teacher, obtain a copy of the diagrams and worksheet for this Explore. Look at diagram 1.
 - Determine the slope of line segment OA. Record your answer in the chart.
 - Evaluate the tangent ratio of $\angle AOB$. Record your answer.
- Repeat step 7 for the remaining four diagrams.
- Reflect**
 - Look back at your completed chart. What can you conclude?
 - How can you determine the measure of an **angle of elevation** if you know the slope of the line? Explain, using an example.
- Extend Your Understanding** For a pilot to land a plane safely, the angle of elevation of the plane must be approximately 3° .

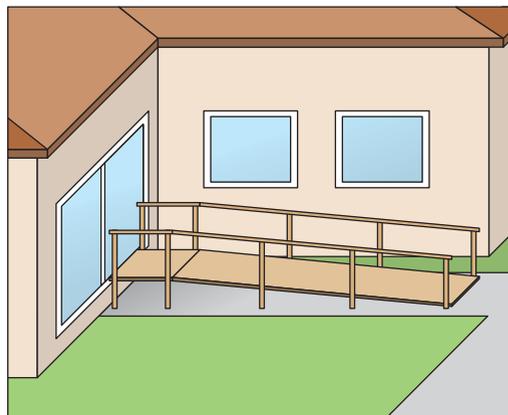


- What is the slope of the plane's landing path? Express your answer as a decimal to the nearest hundredth, and as a percent.
- Express the height of the plane as a percent of the length of the runway. **Hint:** Use the slope formula, with the slope as a percent.
- If the runway is 23.3 km long, what should be the height of the plane when it gets to the runway?

On the Job 1

Work With Slope and Angle of Elevation

Yanick and Emily plan to build a wheelchair ramp for their grandmother. They learn that outdoor ramps must have a slope close to 1 : 12 but not greater. This ensures that the person in the wheelchair can safely travel up and down the ramp.



F.Y.I.

An outdoor wheelchair ramp generally has a slope of 1 : 12. An indoor wheelchair ramp can have a slope of 1 : 9.

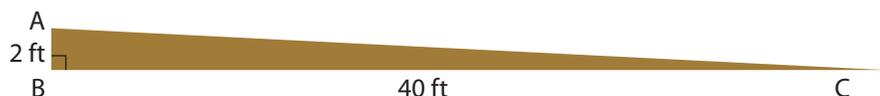
- a) To start, they measure the space they have to build the ramp.
- The distance from the doorway straight down to the ground is 2 feet.
 - The walkway along the ground is 40 feet.

Do Emily and Yanick have enough space for a ramp that meets the safety rule?

- b) Once the ramp is built, what angle will the ramp make with the ground? Express your answer to the nearest degree.

Solution

Draw and label a diagram.



- a) Determine the slope of the ramp. Then, compare it to the maximum slope allowed.

$$\begin{aligned}\text{Slope of Ramp} \\ \text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{40} \\ &= 0.05\end{aligned}$$

$$\begin{aligned}\text{Maximum Safe Slope} \\ \text{Slope} &= \frac{1}{12} \\ &= 0.083\dots\end{aligned}$$

$$0.05 < 0.08$$

They have enough space to build the ramp.

b) Determine the measure of $\angle C$.

$$\tan C = \text{slope of ramp}$$

$$\tan C = 0.05$$

$$\angle C = \tan^{-1}(0.05)$$

$$\angle C = 2.862\dots$$

Substitute the value for slope you found in part a).

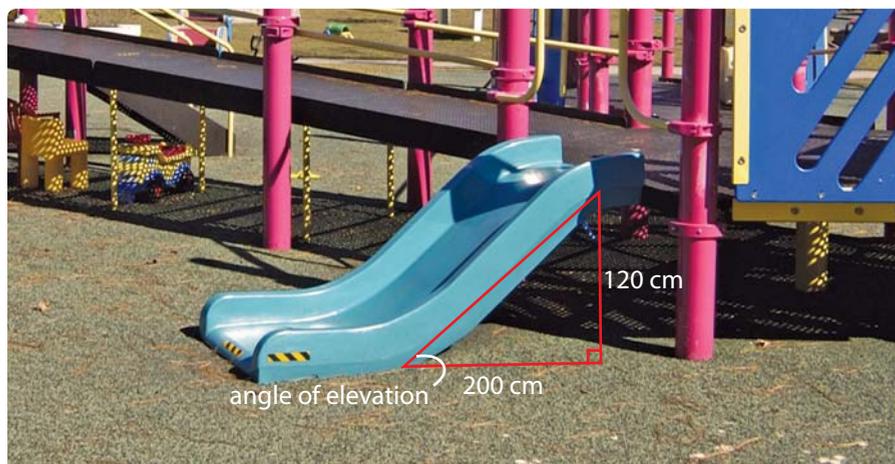


C 2nd TAN⁻¹ 0.05 =
2.862405226

The ramp makes an angle of approximately 3° with the ground.

Your Turn

A local playground has a slide that reaches 200 cm along the ground. The top of the slide is 120 cm above the ground.

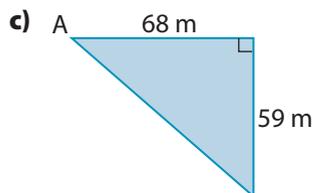
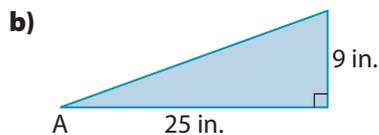
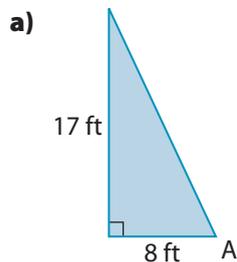


- Safety rules state that the slope of a slide cannot exceed 0.577. Does this slide comply with the rule? Explain.
- What angle does the slide make with the ground? Express your answer to the nearest degree.
- Safety rules state that the maximum angle of elevation of a slide is 30° . What is the difference between the maximum angle and this slide's angle of elevation?

Check Your Understanding

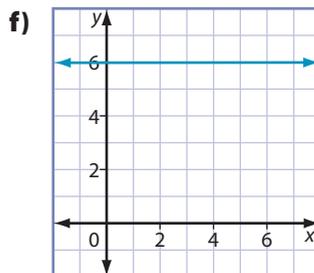
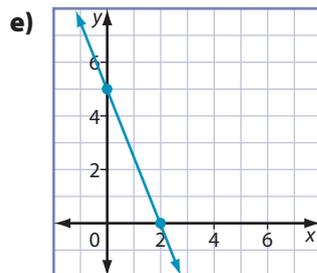
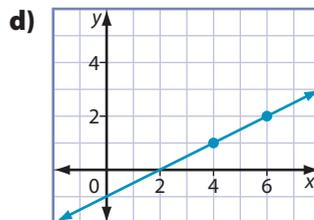
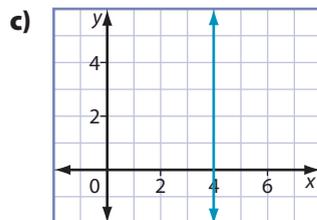
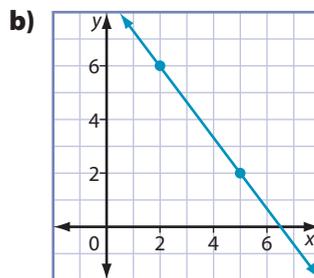
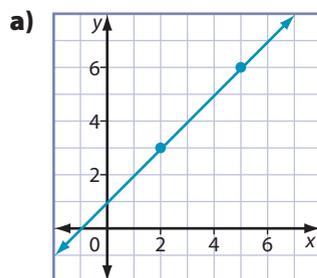
Try It

1. Determine the tangent ratio of $\angle A$ in each triangle.



2. For each triangle in #1, determine the measure of $\angle A$ to the nearest degree.

3. For each graph, what is the measure of the angle of elevation to the nearest degree?

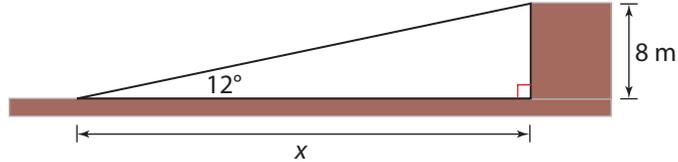


F.Y.I.

Some spaces do not allow a wheelchair ramp to be built with a slope of 1:12. The ramp might need to be built winding from side to side to meet the safety rule. Sometimes permission is given for a ramp to have a maximum slope of 1:8.

Apply It

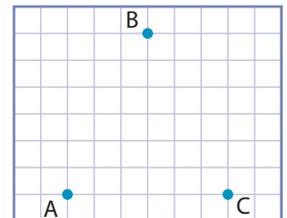
4. What angle does a ramp with a slope of 1:8 make with the ground? Express your answer to the nearest degree.
5. a) An underground parking ramp has been designed to rise at an angle of 12° . The parking lot is 8 m below ground. What is the run of the ramp to the nearest tenth of a metre?



- b) Would this make a good wheelchair ramp? Explain.
6. a) Indoor ramps have a steepness of 1 cm of rise for every 9 cm of run. For an indoor ramp to rise 90 cm, how far along the ground should it run?



- b) What is the angle of elevation of the ramp? Express your answer to the nearest degree.
7. a) What type of triangle is formed by the points A, B, and C on the grid? How do you know?
- b) Determine the slope of line segments AB, BC, and AC.
- c) What can you conclude about the slopes of the two equal sides in this triangle?
- d) Without using a protractor, determine the measure of $\angle A$. Express your answer to the nearest degree.
- e) What can you conclude about the measure of $\angle C$? Explain.



On the Job 2

Slope as a Percent Grade

Tanya works for the department of transportation. Her job is to place warning signs in areas along the highway that may be dangerous.

- What is the purpose of this road sign?
- What does 8% **grade** mean?
- Express the slope of the road as a fraction.
- Explain what the 8% means in terms of the road.
- Tanya inspects a section of the Trans-Canada Highway. She calculates that the road drops 5 m for every 40 m horizontal distance. What is the slope of the road? Explain what this means in terms of the road.
- Warning signs are usually placed in areas where the grade is 6% or greater. Should Tanya install a warning sign along the section of the highway in part e)? If so, what grade should the sign show?



grade

- the slope of a road or railway track
- usually expressed as a percent

Web Link

The Trans-Canada Highway starts/ends in the east, in St. John's, NL. For information about the length of the Trans-Canada Highway or its routes, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

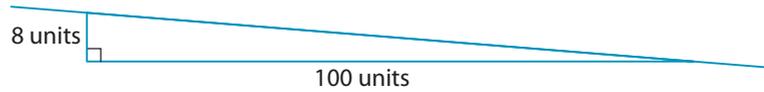
Solution

- The sign warns drivers that they are approaching a steep hill or decline. They should lower their speed. Large trucks with loads should use a lower gear.
- Eight percent grade means the slope of the road is 8%.

$$\begin{aligned} \text{c) } 8\% &= \frac{8}{100} \\ 8\% &= \frac{2}{25} \end{aligned}$$

The slope of the road in lowest terms is $\frac{2}{25}$.

- d) The 8% means that the road drops 8 units vertically for every 100 units horizontally.



The rise and run are measured in the same unit. So, the road could drop 8 metres for every 100 metres horizontally, or it could drop 8 feet for every 100 feet horizontally.



e)
$$\begin{aligned} \text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{5}{40} \\ &= \frac{1}{8} \end{aligned}$$

The slope is $\frac{1}{8}$. The road drops 1 m vertically for every 8 m horizontally.

- f) To determine the grade of the road, convert the slope to a percent.

$$\frac{1}{8} = 12.5\%$$

$12.5\% > 6\%$. This is a very steep hill. Tanya should install a warning sign that reads 13%.

Your Turn

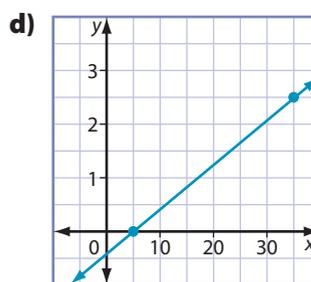
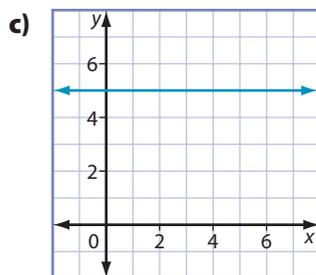
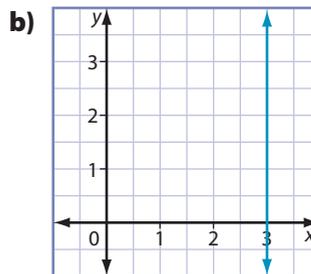
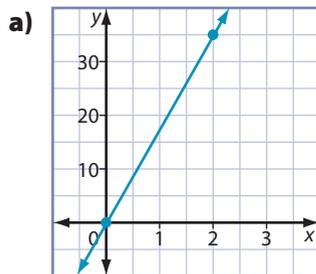
- What does this sign warn motorists about?
- Express the slope of the road as a fraction.
- Explain what the 18% means.
- A road drops 5 ft for every 90 ft of horizontal distance. Would you expect to see a warning sign there? Explain why or why not.



Check Your Understanding

Try It

1. Determine the slope of each line.



2. a) Suppose each line in #1 represents a road. Which one would be the easiest to travel?

b) Which one would be impossible to travel? Why?

3. a) Explain the meaning of the sign shown.

b) Write the slope of the road as a fraction.



4. The table shows details of the grades of various roads. Copy and complete the table. Express slope as a decimal to the nearest hundredth.

Road Name	Rise	Run	Slope as a Fraction	Slope as a Decimal	Percent Grade
Rarely Driven Route	1500	5000			
Snail Pace Strip	9	42			
Pothole Path				0.05	
Maniac Motorway			$\frac{17}{90}$		
Hurry-Up Highway			$\frac{3}{50}$		
Traffic Jam Thoroughfare					1%
Reckless Ramp				0.5	
Boggy Boulevard					6.25%

F.Y.I.

Warning signs are usually placed where the grade of the road is 6% or greater.

**F.Y.I.**

One third of all reports of people falling from a height involve ladders. Many of these injuries are caused by incorrect use of the ladder.

5. Determine the angle of elevation of a road with each grade. Express your answers to the nearest degree.

- a) 16% b) 2%
c) 6% d) 20%

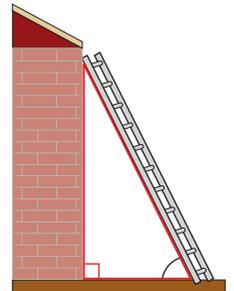
Apply It

6. Refer to the table you completed in #4. Which of the roads require a warning sign?
7. Jason wants to know the grade of his new driveway. He measures that the driveway rises 5' over 50' along the ground. What is the grade of his driveway?
8. What is the grade of a wheelchair ramp that rises $1\frac{1}{2}$ ft over a horizontal distance of 18 ft? Express your answer to the nearest percent.



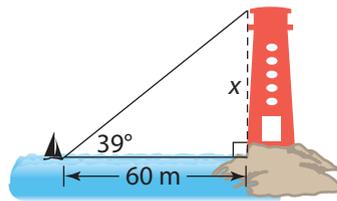
9. Tyler is helping his mother paint the eavestrough on their house. He leans the ladder on the bricks 4 m above the ground. The bottom of the ladder is 2 m away from the house.

- a) Make a sketch similar to the one shown. Include the given dimensions.
- b) Safety guidelines state that the maximum slope of a ladder should be 4. What is the angle of elevation of a ladder placed at this slope? Round your answer to the nearest degree.
- c) Does Tyler have the ladder positioned safely? Explain.

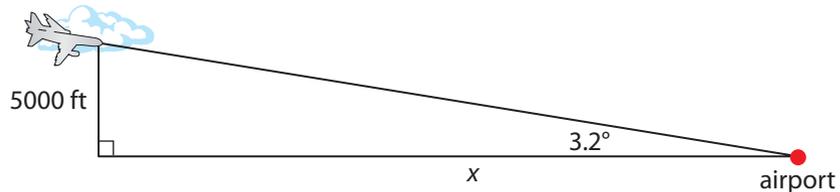


Work With It

- The bottom of a ladder is 2 m from a house. The angle between the ladder and the ground is 65° .
 - How far up the house does the ladder reach? Round your answer to the nearest tenth of a metre.
 - What is the length of the ladder? Round your answer to the nearest tenth of a metre.
- From a boat 60 m away, the angle of elevation of the top of a lighthouse is 39° . Determine the height of the top of the lighthouse from sea level. Express your answer to the nearest tenth of a metre.



- The recommended slope of a curb ramp is 1 : 10. The Newfoundland and Labrador Credit Union built a new office in Gander. To make the building wheelchair accessible, there needs to be a ramp from the parking lot to the sidewalk.
 - If the sidewalk is 6" above the parking lot, what must be the length of the run of the ramp in order to comply with the 1 : 10 ratio?
 - What angle does the ramp make with the ground?
- A plane is flying at an altitude of 5000 feet. An air traffic controller at St. John's International Airport measures the angle of elevation to be 3.2° . What is the plane's horizontal distance to the airport? Express your answer to the nearest foot.



- Find examples of things that have a slope of zero and an undefined slope.

Discuss It



- 6.** Carpenters must follow guidelines when building stairs. The rule is that for every 250 mm of run there should be a 200-mm rise. Chad is building a set of stairs. Each tread will have a run of 10 inches and a rise of 8 inches. Will his stairs meet the requirements? Explain.
- 7.** Describe a road with a 0% grade. State some effects this would have on driving.
- 8. a)** Why is the slope of a horizontal line zero? Draw a picture to help explain your answer.
- b)** Why is the slope of a vertical line undefined? Include a drawing in your answer.

- 9.** Examine one end of the frame of the swing set. How are the slopes of the two poles that form a triangle with the ground related? Why is this important?



- 10. a)** What does the percent on the sign mean?
- b)** Explain why drivers should use a low gear on this part of the road.



- 11.** Andrea determines the angle of elevation of the line on the graph. Her work is shown below.

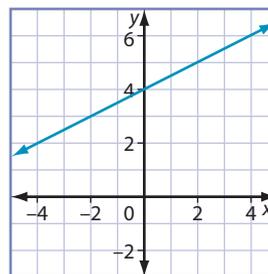
$$\begin{aligned} \text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2 - 0}{5 - 4} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

$$\tan(\text{angle of elevation}) = \text{slope}$$

$$\tan(\text{angle of elevation}) = 2$$

$$\tan^{-1}(2) = \text{angle of elevation}$$

$$0.035 \approx \text{angle of elevation}$$



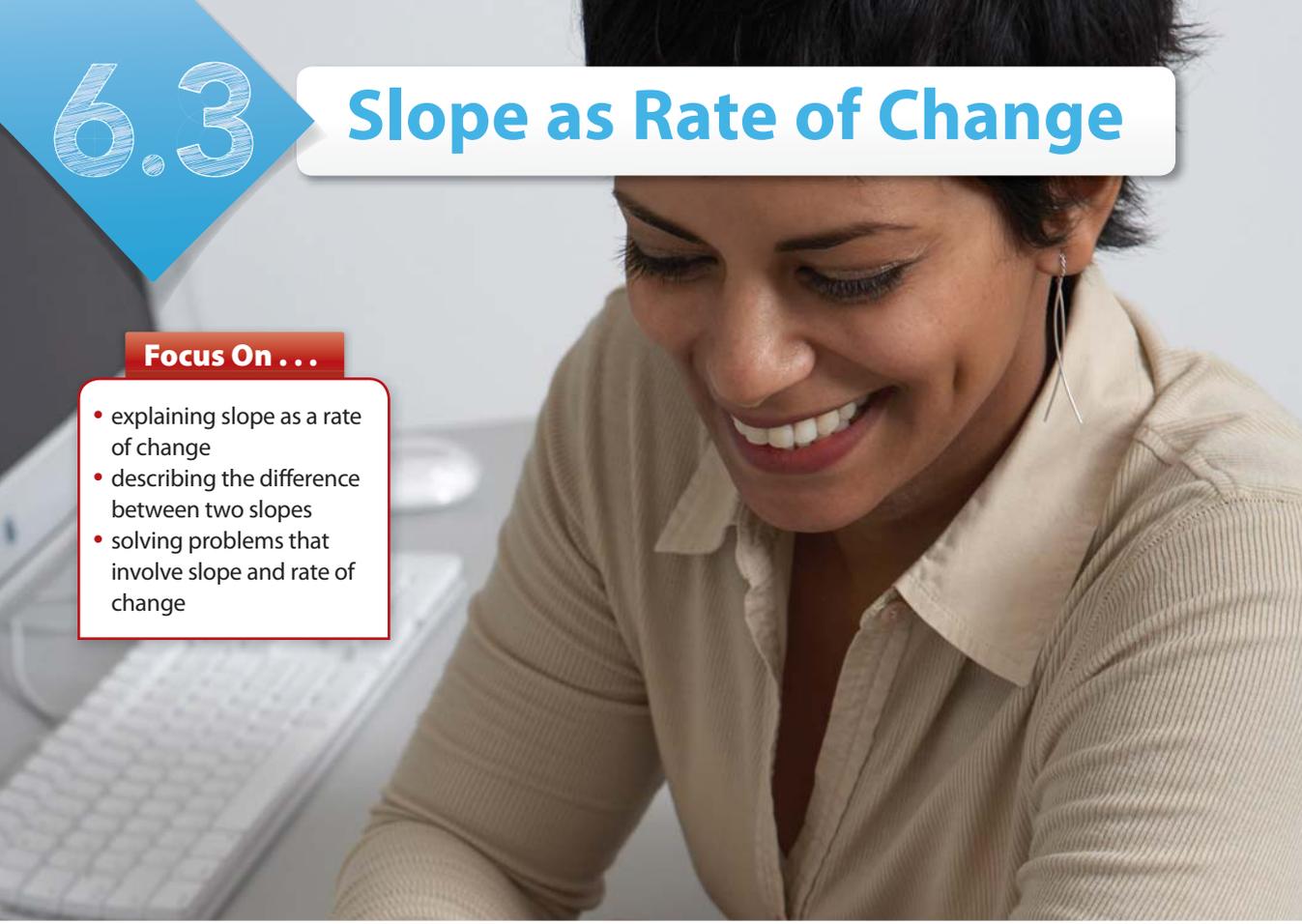
- a)** At which step would Andrea realize she had made her first error? Why?
- b)** Identify and correct the errors in Andrea's solution.

6.3

Slope as Rate of Change

Focus On ...

- explaining slope as a rate of change
- describing the difference between two slopes
- solving problems that involve slope and rate of change



rate of change

- a change in one quantity relative to the change in another quantity

Materials

- toothpicks
- grid paper 
- ruler

*When you earn a wage, the money you receive is related to the number of hours you worked. How could you represent this **rate of change**? Can you think of other rates of change?*

Explore Rate of Change

- Toothpicks are arranged to make the pattern shown.



- Copy the table and extend it to 8 squares. Make the patterns and record the number of toothpicks used. The first one is done for you.

Number of Squares	Total Number of Toothpicks
1	4
2	
3	

- b) Determine the change in the number of squares.

Determine the difference between each pair of values in the first column.

Number of Squares	
+1	1
+1	2
	3

The change in the number of squares is always 1.

- c) Determine the change in the total number of toothpicks. What do you notice?

Determine the difference between each pair of values in the second column.

2. Suppose you make the pattern using each of the following number of toothpicks. If you add one more square to each pattern, what will be the total number of toothpicks?
- a) 34
b) 97
3. a) On grid paper, plot the data in the table of values. What shape do the points form?
b) Choose any two points. Determine the rise and run. What is the slope of the line?
4. **Reflect**
- a) How do your answers to steps 1b) and c) relate to the slope of the line?
b) Explain why this relationship applies to all lines.
5. **Extend Your Understanding** Determine the rate of change in the total number of toothpicks for a pattern of triangles. Use the relationship from step 4.

Number of Triangles	Total Number of Toothpicks
1	3
2	5
3	7

Puzzler

Suppose you continue the square toothpick pattern in step 1 of the Explore. How many squares can you make with 40 toothpicks?

On the Job 1

Calculate Rate of Change

One year ago, Becky started working at a grocery store. She put \$100 per month into her bank account. Becky already had \$200 in the account. The table shows the growth in Becky's bank account.

Strategy



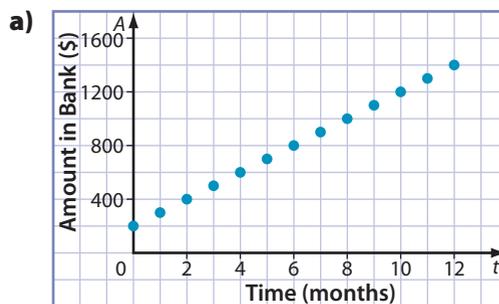
**Make a
Systematic
List**

Time (months)	Amount in Bank Account
0	\$200
1	\$300
2	\$400
3	\$500
4	\$600
5	\$700
6	\$800
7	\$900
8	\$1000
9	\$1100
10	\$1200
11	\$1300
12	\$1400



- On grid paper, plot the data in the table.
- Determine the slope of the line that would connect the points.
- What is the rate of change in Becky's savings from month 1 to month 4? from month 5 to month 12? Explain.
- What do you notice about the rate of change for each time period in part c)?

Solution



Strategy

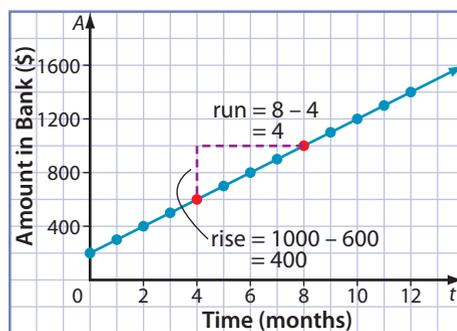


Develop a Strategy

How else could you determine the rise and run?

- b) The slope of the line is constant. So, Becky can use any two points on the line. She uses the two points in red.

$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{400}{4} \\ &= 100\end{aligned}$$



The slope of the line is 100.

- c) Rate of change = $\frac{\text{vertical change}}{\text{horizontal change}}$

Month 1 to Month 4

$$\begin{aligned}\text{Rate of change} &= \frac{600 - 300}{4 - 1} \\ &= \frac{300}{3} \\ &= 100\end{aligned}$$

Month 5 to Month 12

$$\begin{aligned}\text{Rate of change} &= \frac{1400 - 700}{12 - 5} \\ &= \frac{700}{7} \\ &= 100\end{aligned}$$

Determine these values using the table or the graph.

The rate of change has a constant value of 100. This means the amount increases by \$100 as the month increases by 1.

- d) The rate of change is equal to the slope of the line on the graph.

Your Turn

Adam picks blueberries to be sold in a local market. The table shows Adam's earnings for 1 to 5 boxes of blueberries.

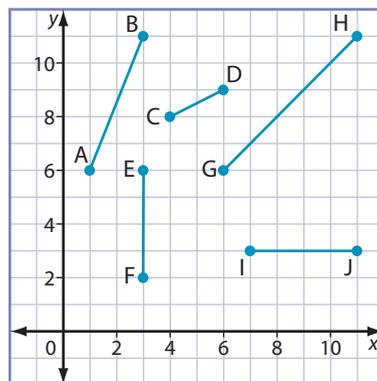
Number of Boxes Sold	Adam's Earnings (\$)
1	3
2	6
3	9
4	12
5	15

- On grid paper, plot the data. What do the points form?
- How does the number of boxes sold affect Adam's earnings?
- Determine the slope of the line that would connect the points on the graph.
- Determine the rate of change of Adam's earnings. What does this rate of change mean?

Check Your Understanding

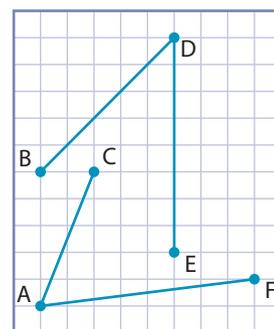
Try It

1. **a)** Determine the rise and run of each line segment.
- b)** Determine the slope of each line segment.



2. Determine the slope of each line segment.

- | | |
|--------------|--------------|
| a) AC | b) ED |
| c) BD | d) AF |



3. Examine the table of values.

- a)** What is the change in the x -values from one row to the next?
- b)** What is the change in the y -values?
- c)** What is the slope of the line that would connect these points on a graph?

x	y
0	4
1	6
2	8
3	10
4	12

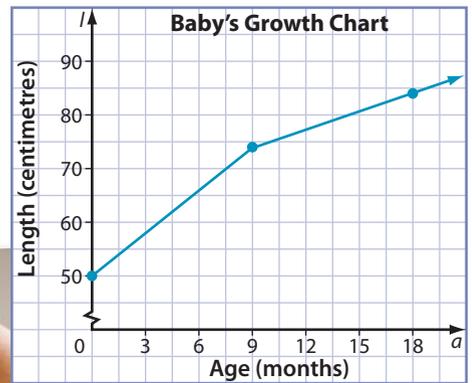
Apply It

4. Josh drives non-stop from Deer Lake to Holyrood. His average speed is 90 km/h. The table shows the distance he drove during each hour of the trip.

Time (h)	Distance (km)
1	90
2	180
3	270
4	360
5	450
6	540

- a)** Graph the data. Connect the points with a straight line.
- b)** What is the slope of the line?
- c)** How is the slope related to the rate of change in distance?

5. A baby's growth is plotted for the first 18 months of her life. Determine the rate of change in growth from age 9 months to age 18 months.



6. Julie earns \$20 for babysitting for 2 h.
a) Copy and complete the table.

Time Worked (h)	Amount Earned (\$)	Rate of Change
0	0	
1	10	$10 - 0 = 10$
2	20	
3		
4		
5		

- b) What is the rate of change in Julie's earnings? Explain what this means.
c) Suppose Julie gets an increase of \$5 for 2 h. What is her new rate of change in earnings?
d) Graph both of Julie's earnings on one grid. What is similar about the graphs? Explain.
e) Describe any differences between the two graphs.

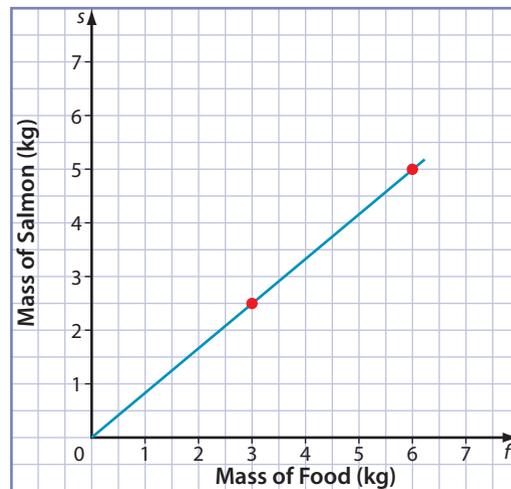
On the Job 2

Solve a Problem Involving Rate of Change

Farmed fish eat pellets made of fishmeal and fish oil.



The graph shows the approximate relationship between the mass of dry food pellets eaten by a salmon and the mass of the salmon.



- Determine the rate of change in the mass of salmon.
- Describe the relationship between the mass of a salmon and the mass of food it consumes.
- Suppose fish farmers gather data on this relationship regularly. Which of the following would a fish farmer prefer to see? Justify your answer.
 - a graph with a steeper slope
 - a graph with a less steep slope

F.Y.I.

Farmed salmon continue to gain weight until they reach maturity around 6 kg.

Strategy

Eliminate Possibilities

Solution

- a) The rate of change is equal to the slope. Choose two points that are on the line segment. For example, the points in red.

$$\begin{aligned} \text{Rate of change} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{5 - 2.5}{6 - 3} \\ &= \frac{2.5}{3} \\ &= 0.833\dots \end{aligned}$$

Choose points for which you can easily read the values.

The rate of change in mass of salmon is approximately 0.8.

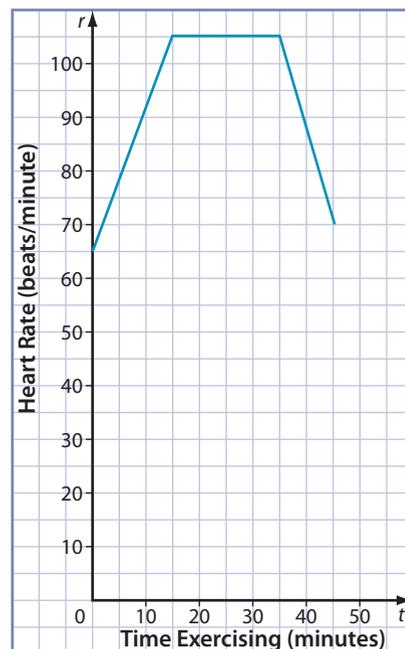
- b) As the salmon consumes food, it grows at a steady rate. For every kilogram of food consumed, the salmon gains 0.8 kg of body weight.
- c) • A steeper slope means the salmon are gaining more weight for each kilogram of food they consume.
• A less steep slope means they are gaining less weight for each kilogram of food consumed.

Fish farmers would prefer the steeper slope because they would not need to buy as much dry fish food.

Your Turn

Nicole records her heart rate during an aerobics class. Her results are shown in the graph.

- a) When is Nicole's heart rate increasing? How do you know?
- b) What is the rate of change in heart rate for this interval?
- c) Describe this rate of change in words.
- d) For 20 minutes during the class, Nicole's heart rate did not change. What was her heart rate during this interval?
- e) How do you know it did not change?
- f) Determine the rate of change in heart rate for this interval.



Check Your Understanding

Try It

1. Determine the slope of the line in a graph of the data.

x	y
0	1
2	4
4	7
6	10
8	13

2. Does a graph of the data in each table show a constant slope? Explain.

Table A

x	y
0	4
1	6
2	8
3	10
4	12

Table B

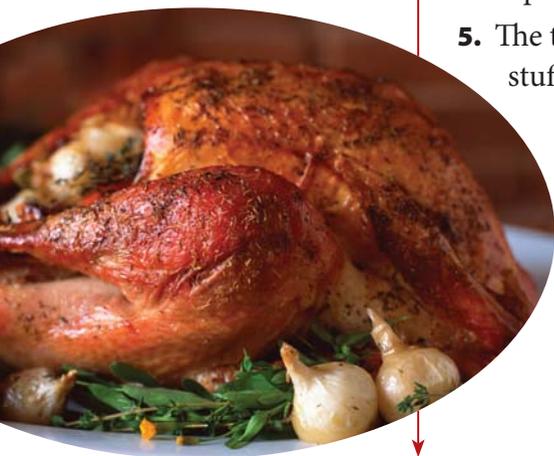
x	y
0	3
1	4
2	6
3	9
4	13

Apply It

3. Suppose Toshi drives at a constant rate for 4 h. After 1 h, he has travelled 100 km. After 3 h, he has travelled 300 km. Determine the rate of change in distance, and explain what it means.
4. Olivia babysits for the Youngs and always receives the same rate of pay. She babysat for 2 hours and received \$16. She was paid \$40 for 5 hours. Determine the rate of change in pay, and explain what it means.
5. The table shows the time to cook stuffed turkeys of varying sizes.

- a) Graph the data.
- b) Determine the slope of the line. Explain what it means.

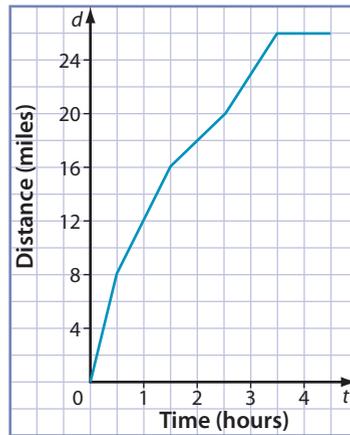
Mass (kg)	Time (h)
3	3.5
5	4
7	4.5
9	5



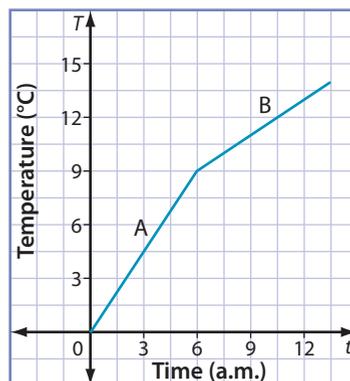
F.Y.I.

There have been over 60 runnings of the Huffin' Puffin Marathon, which used to be called the Newfoundland Provincial Marathon. It is the oldest marathon in Canada and the third oldest in North America.

6. Chelsea plans to run in the Huffin' Puffin Marathon. The results of her last marathon are shown in the graph.

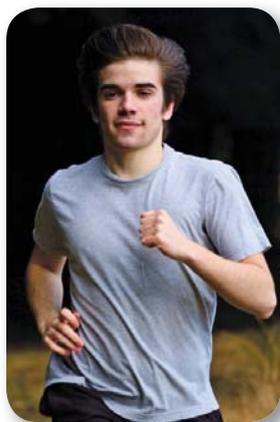


- Identify the intervals on the graph that have constant slope.
 - Determine the slope of each of these intervals.
 - Explain the slope as a rate of change.
 - During which interval did the distance Chelsea ran not change? Explain what this means.
 - What is the rate of change for this interval?
7. The graph shows the hourly temperature for the morning.

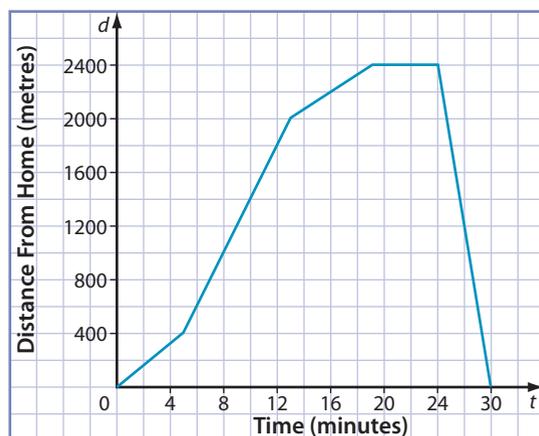


- Determine the slopes of intervals A and B.
- How are the slopes the same? How are they different?
- Describe the slope of interval A as a rate of change.
- Describe the slope of interval B as a rate of change.

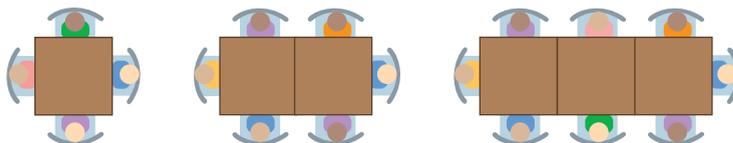
Work With It



1. The graph shows the distance Jim travels during a 30-min run.



- a) Why are some segments of the graph steeper than others?
 - b) Calculate the rate of change in distance for each segment.
 - c) What does it mean when the rate of change is zero?
2. A classroom has small square tables that can sit one person on each side. The teacher wants to seat more than four students together. So, she puts several tables together.



- a) Copy and complete the chart to show the number of people seated at each set of tables.

Number of Tables	Number of People
1	
2	
3	
4	
5	

- b) How does the number of people seated change as the number of tables increases by 1?
- c) What would be the slope of the line on a graph of the data?
- d) Describe the slope as a rate of change.

F.Y.I.

The photo shows the Wooden Roller Coaster in Vancouver, BC.

Web Link

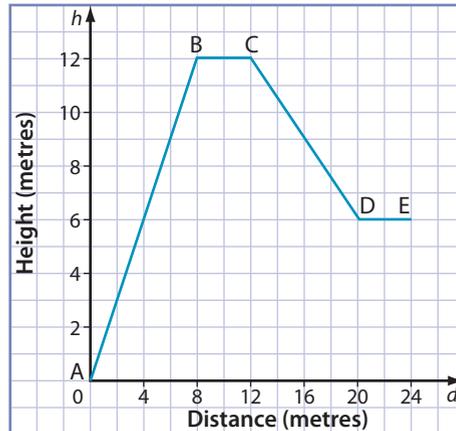
For information on how the Wooden Roller Coaster was built or to see a video of the ride, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

Strategy



Work
Backward

3. The graph represents the side view of part of a roller coaster.



- a) Determine the rate of change in height for each segment of the graph. Explain the meaning of each rate of change.
b) Which part of the ride is the steepest?

Discuss It

4. Explain the meaning of rate of change. Use an example.
5. a) The roof of a house has a pitch of $\frac{5}{12}$. Explain what this means in terms of rate of change.
b) How does a roof pitch of $\frac{5}{12}$ differ from a pitch of $\frac{12}{5}$?



What You Need to Know

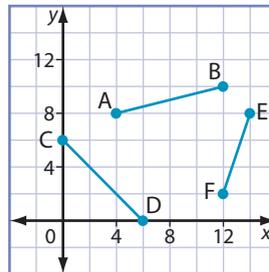
Section After this section, I know how to . . .

- 6.1**
- explain slope as rise divided by run
 - verify that the slope is constant for a straight object
 - solve problems that involve slope
 - describe contexts that involve slope
- 6.2**
- describe the conditions for a slope that is zero or undefined
 - explain the relationship between slope and angle of elevation
 - explain the safety implications of different slopes
 - solve problems by applying the slope formula
- 6.3**
- describe the difference between two slopes
 - explain slope as a rate of change
 - solve problems that involve slope and rate of change

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

6.1 What Is Slope?, pages 262–273

1. Determine the slope of each line segment.

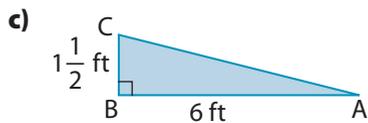
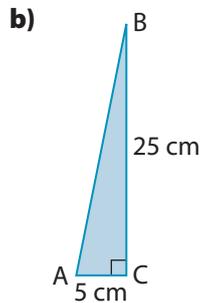
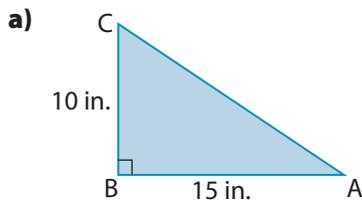


2. Copy and complete the table.

	Rise	Run	Slope
a)	0	6 m	
b)	2 in.	3 in.	
c)	2 m	200 cm	
d)	4 m	0	
e)	60 cm	3 m	

6.2 Relationship Between Slope and Angle of Elevation,
pages 274–285

3. For each triangle, determine the measure of $\angle A$ to the nearest degree.



4. A bike ramp has a slope of 1 : 2. What angle does the ramp make with the ground, to the nearest degree?

5. Determine the angle of elevation of a road with the following grades. Express your answer to the nearest degree.

- a) 0% b) 5% c) 17%

6.3 Slope as Rate of Change, pages 286–297

6. Compare the upper and lower roofs shown.

- a) Describe the difference in the appearance of the roofs.
 b) Which roof would be the most difficult to walk on? Why?
 c) Suppose the pitches of the roofs are $\frac{6}{12}$ and $\frac{12}{6}$. What is the pitch of the upper roof? What is the pitch of the lower roof? Explain your reasoning.



7. a) Graph the data in the table.

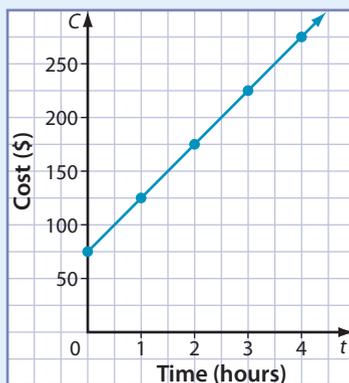
x	0	1	2	3	4
y	1	4	7	10	13

b) Describe the slope.

For #1 to #3, select the best answer.

1. What is the slope of the line?

- A 50
 B $\frac{250}{3}$
 C 4:2
 D $\frac{2}{50}$



2. For which table does the graph have a constant slope?

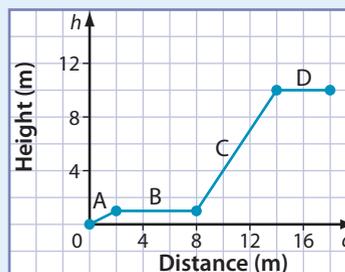
A	x	y	B	x	y	C	x	y	D	x	y
	1	1		0	2		0	5		0	3
	2	2		2	4		1	10		1	6
	3	4		4	6		2	15		2	9
	4	7		6	8		3	20		3	12
	5	11		8	10		4	30		5	15

3. If each table of values is graphed, which line would have the greater slope?

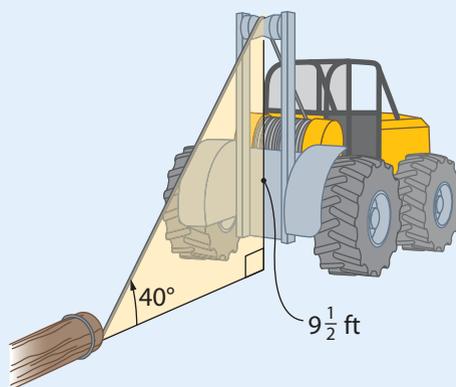
A	x	y	B	x	y	C	x	y	D	x	y
	0	2		0	20		0	60		0	100
	1	8		1	21.5		1	65		1	101
	2	14		2	23		2	70		2	102
	3	20		3	24.5		3	75		3	103
	4	26		4	26		4	80		4	104

4. The graph represents the height versus distance for part of a roller coaster.

- a) Determine the slope of each segment shown.
 b) Describe each slope.



5. Kayden operates a cable skidder, which pulls logs up a hill to a landing. The top of the cable is $9\frac{1}{2}$ ft high and the cable has an angle of elevation of about 40° . How long is the cable, to the nearest foot?



6. A town has a new playground. The top of the slide is 2 m higher than the bottom. Its horizontal distance along the ground is 300 cm.



- a) Safety regulations state that the slope of a slide must not exceed 0.577. Will this slide pass an inspection?
- b) What horizontal distance meets the requirements for a slide that is 2 m high? Express your answer to the nearest centimetre.
7. An airplane approaching St. John's International Airport is flying at an altitude of 2600 m. Its horizontal distance from the airport is 48 km.
- a) Determine the airplane's angle of elevation to the nearest degree.
- b) The safe angle of elevation for large planes is 3° . Does this plane have a safe angle of elevation?
8. Maura earned \$18 per hour when she started her job. After 6 months, she received an increase of \$2 per hour. Six months later, she received another increase of \$2 per hour.
- a) Graph Maura's earnings over the last 12 months. Does the graph show a constant rate of change in earnings? Explain.
- b) Determine the slope of the line that represents each of Maura's salaries.
- c) Describe each slope as a rate of change.



Tools of the Trade

Loggers use chainsaws attached to machines called harvesters to cut down, or fell, trees. They delimb the trees and leave some parts in the forest to promote new growth. Then, loggers remove the trees using skidders or other heavy machinery. For more information on logging, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

F.Y.I.

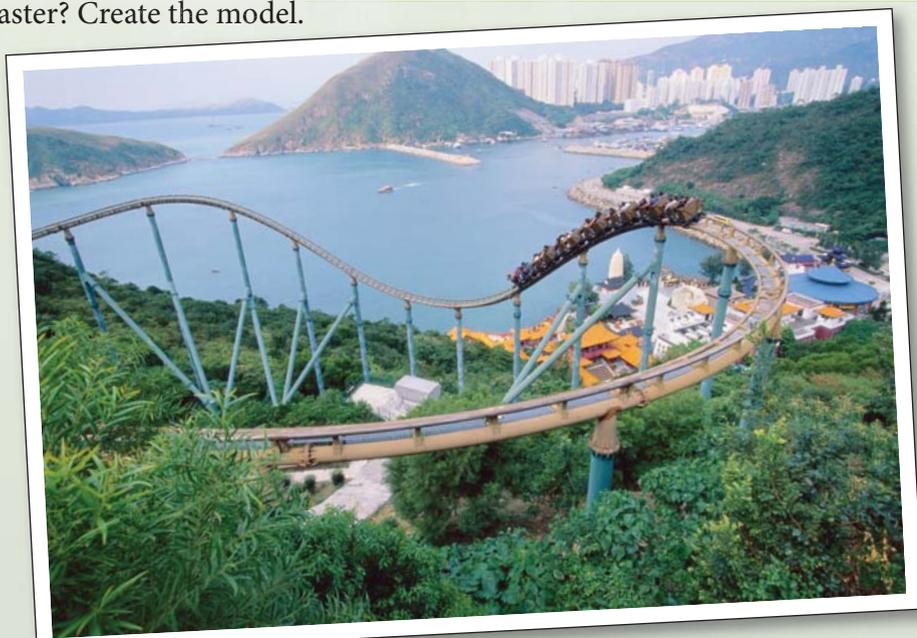
Amelia Earhart was the first woman to fly solo across the Atlantic Ocean. On May 20, 1932, she departed from Harbour Grace, NL, and she landed safely in Northern Ireland about 15 h later.



Design a Coaster Ride

Design and create a model of a mountain coaster ride. The ride zigzags across the face of the mountain.

1. Design a model of a coaster ride using the following guidelines:
 - The face of the mountain in the model is in the shape of a 12-cm by 12-cm square.
 - The ride must start at a top corner and end at the bottom corner on the opposite side.
 - The ride must include 3 to 6 segments.
 - The slope of each segment of the ride must be 20% or less.
2. What is the maximum angle of elevation for each segment of the coaster?
Hint: Make sure the slope is 20% or less.
3. Include a sketch of the model on grid paper, with labels showing the dimensions. Include a calculation of the slope of each segment.
4. Determine the total length of the coaster in the model.
5. What materials will you use for the model of the mountain face and the coaster? Create the model.



Slippery Slope

Play in a group of 2 to 4 players.

1. The dealer is the person whose birthday is next. The dealer shuffles the cards and deals two cards to each player, face up.
2. Cards have their face value. Aces have a value of one.
3. The first card dealt represents the rise. The second card represents the run. Each player calculates the slope represented by their cards.
4. The person with the steepest slope scores one point.
5. The first person to score ten points wins.
6. If there is a tie for the steepest slope, both players get a point.
7. If you run out of cards before any player scores 10, shuffle the deck and continue to play.

Materials

- deck of cards with face cards removed



Other Ways to Play

- A** Keep the face cards in the deck.
- Jacks have a value of 11.
 - Queens have a value of 12.
 - Kings have a value of 13.
- B** Once each person calculates their slope, the person with the steepest slope gets all of the cards for that round. The person with the most cards at the end of the game wins.
- C** Each player determines their own slope and keeps a running total of their values. The person with the greater sum wins.
- Make up and play your own version of Slippery Slope!